University of Arkansas at little Rock
Department of Systems Engineering

SYEN 4399/5399 Estimation Theory
Fall 2008

MIDTERM EXAM
Thursday, October 23, 2008

- This is a closed book exam.

- Calculators are not allowed.

- There are 5 problems in the exam.

- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.

- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and show all relevant work. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.

- Please be neat and box your final answer, we cannot grade what we cannot decipher.

Name

Problem 1

What is

(1) an unbiased estimator?

(2) an efficient estimator?

(3) a MVU estimator?

(4) a BLUE estimator?

(5) a white noise

Problem 2

The data $x[n] = Ar^n + w[n]$ for $n = 0, 1, \cdots, N - 1$ are observed, where $w[n]$ is WGN with variance σ^2 and $r > 0$ known .

(1) Find the CRLB for A

(2) Show that an efficient estimator exists and find its variance (using the CRLB theorem).

(3) Write the linear model for this data in the form $\mathbf{x} = HA + \mathbf{w}$.

(4) Using (3), find the MVU estimator of A. What is its variance?

(5) Find the BLUE estimator of A, and its variance.

(6) Compare the 3 estimators. Explain!

Problem 3

Consider the problem of line fitting: given the observations

$$x[n] = A + Bn + w[n], \quad n = 0, 1, \cdots, N - 1.$$

where $w[n]$ is WGN. We want to estimate A and B. Hence, we have a vector parameter $\theta = \begin{pmatrix} A \\ B \end{pmatrix}$

(1) Is it a linear model? If yes, write it in vector form $(\mathbf{x} = H\theta + \mathbf{w})$.

(2) Find the MVU estimators of A and B (simplify the formula).

(3) Are the estimators found in (2) efficient? Find their variances.

Problem 4

Consider the problem of a DC level in white noise:

$$x[n] = A + w[n], \quad n = 0, 1, \cdots, N-1.$$

where $w[n]$ is white noise with variance σ^2 (*and of unspecified pdf*), then the problem is to estimate A. Since $w[n]$ is not necessarily Gaussian, we cannot use the MVU estimator formula [1]. Find the BLUE estimator of A.

<u>Hint</u>: Because $E[x[n]] = A, s[n] = 1$, and therefore $\mathbf{s} = \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$

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[1]Recall that the MVU estimator applies to a linear model with WGN.